**Fig.1.** Instantaneous frequency $\dot{\varphi}(t)$ used for generation of simulated signal (top). Generated signal (bottom). Blue – whole signal $x(t)$. Green – simulated shaft-related component $A(t)\sin\left(2\pi \int_0^t \dot{\varphi}(t)dt\right)$. Red – noise component $n(t)$.

After transformation of signal $x(t)$ to $z_\varphi(\varphi)$ we observe that component related to instantaneous frequency $\dot{\varphi}(t)$ manifests periodic behavior. Fig. 2 presents resulting normalized signal $z_\varphi(\varphi)$ and comparison between power spectral densities (PSD) of original signal $x(t)$ and $z_\varphi(\varphi)$.
Fig. 2. Normalized signal $z_{\phi}(\varphi)$ (top). PSD of signal $x(t)$ (bottom/left) and of $z_{\phi}(\varphi)$ (bottom/right). Red and green lines represents respectively upper and lower confidence interval.

Transfer function $G(\theta)$ can now be calculated and applied to the signal $x(t(\varphi)) = x'(\varphi)$, so to the original signal $x(t)$ but observed in fixed-angle domain. In order to obtain separated component related to $\hat{\varphi}(t)$ after convolving $G(\theta)$ with $x'(\varphi)$ we can back-transform resulting signal to the original time domain (usually performed by digital angular resampling).
Fig. 3. Results of applied separation. Blue – original signal $x(t)$. Green – extracted shaft-related component $A(t)\sin\left(2\pi \int_0^t \phi(t) dt\right)$. Red – extracted noise component $n(t)$.

For the presented example $R^2$ value was equal to 0.998. Separation results are presented in figure 3. Note that individual components illustrated fig. 1 and fig. 3 experience evident difference at the time range borders. This imperfection might be resolved by implementation of signal windowing, which is not included in the paper.
3 Laboratory Experiment

The test signals were collected from a test rig equipped with a 0.75 kW motor and 0.75 kW braking motor that introduced the load to the system, parallel gearbox with ratio 2.91, and two spherical rolling element bearings type YAR204-2F. The data was collected with the 4-channel NI card 9233, with sampling frequency 50 kHz and VIS-311B accelerometers with 30 kHz range. The motor and the braking unit were controlled by external software synchronized with the data collecting application, which enabled acquisition of vibration signals corresponding to a predefined speed and load profiles. Exemplary predefined speed profile is demonstrated in fig. 4. During this session, the load was held at a constant 0.33kW, and the torque was in range from 6 Nm to 20 Nm. One of the bearings had introduced a local outer race fault with the characteristic frequency equal to 5.15 relative order.

![Diagram](image_url)

**Fig. 4.** Rotational speed profile used for experiment (top). Measured vibration signal (bottom). Blue – original signal. Red – component related to periodic excitation of resonances and noise. Green – component related...
For clarity of presentation we display obtained results as well as the original signal in the form of spectrograms that give overall view of time-frequency distribution of signal energy (see fig. 5). In the middle panel of figure 5 we can observe clear presence of harmonic components of the frequency related to rotational speed of the test rig. Those are generated by unbalanced shafts and during gear-mesh process. Bottom panel of figure 5 displays residual components of the signal representing excitation of local resonances by faulty rolling element bearing during its’ operation. Lighted areas between 1 and 2kHz as well as around 3.5kHz are constant in time in the sense of their frequency range. However, relation of their amplitudes to the value of rotational speed is observed. Additionally, when rotational speed decreases, between second 12’th and 14’th separate pulses originated from operation of faulty bearing are visible.

The spectrograms presented in fig. 5 enabled (according to authors knowledge for the first time) time-frequency representation of separated shaft-related components and periodic excitations (structural components) with noise for non-stationary signals. The raw spectrogram of the original signal shows a time-varying mixture of different sources without any insight to their time occurrence. On the other hand, the spectrogram of separated shaft-related components clearly shows harmonics of shaft and major gear-meshing components, time function of which perfectly matches the true speed profile illustrated in fig. 4.
Fig. 5. Spectrograms of the original signal (top) extracted shaft-related components (middle) and periodic excitations with noise (bottom).

Finally, the method enabled generation of the spectrogram of periodic excitations with noise with normalized amplitudes, which overcomes the problem of time-varying amplitudes detrimental to cyclostationary analysis.
4 Conclusions

The paper illustrated the idea of a combination of DRS techniques with a novel transform of non-stationary signals enabling separation of separate machine related components. The core idea of the paper is illustrated via spectrograms of separated shaft-related components and periodic excitations (structural components) with noise for signals with significantly varying speed (300%) and varying torque. As illustrated on the test rig signal with introduced REB outer race fault, the proposed technique seems to give outstanding prospects for future applications of fault detection of ma-chinery working in highly non-stationary operating conditions.

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5 References

7. Barszcz T., Decomposition of vibration signals into deterministic and nondeterministic


6 APPENDIX A – Best Paper Award official AGH information

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