period Φ:

$$z_{\Phi}(t) = \frac{x(t)-\mu_{\Phi}(t)}{\sigma_{\Phi}(t)} \quad (2)$$

where $\sigma_{\Phi}(\varphi(t))$ is localized angular standard deviation of *x(t)* for the period Φ:

$$\sigma_{\Phi}(\varphi(t)) = \sqrt{\frac{1}{\Phi}\int_{u=\varphi}^{\varphi+\Phi}(x(u)-\mu_{\Phi}(\varphi(u)))^2\,du} \quad (3)$$

and $\mu_{\Phi}(\varphi(t))$ being a localized angular mean of *x(t)* for the angular period Φ:

$$\mu_{\Phi}(\varphi(t)) = \frac{1}{\Phi}\int_{u=\varphi}^{\varphi+\Phi} x(u)\,du \quad (4)$$

It can be observed that even though the signal is not periodic by itself, a transformation which produces purely periodic component does exist. For generalized angular deterministic signals such transformation is possible by changing the domain of the signal *x(t)* from *t* to $\varphi(t)$, and by applying localized normalization, therefore removing fluctuations of the instantaneous amplitude.

In its simplest form vibration signal generated by rotating shaft can be represented as a pure sinusoidal wave of the frequency equal to rotational frequency of the shaft. Even though in real life vibration signals contain number of harmonics the following model focuses only on the fundamental component to give the general idea of signal generating process.

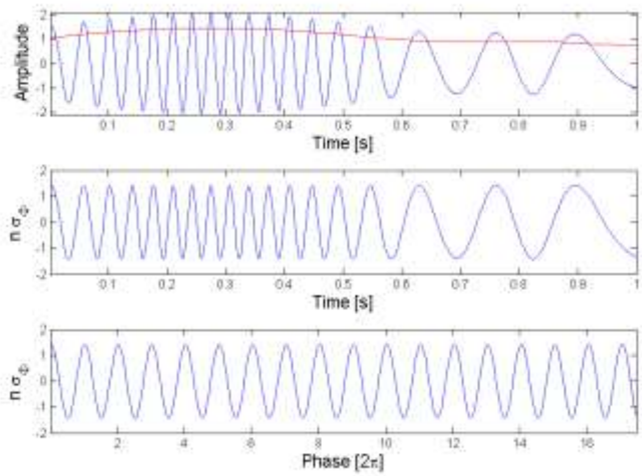

Fig. 3. Top – example of generalized angular deterministic signal x(t) (blue line) and its localized angular standard deviation $\sigma_\Phi(t)$. Center - signal $z_\Phi(t)$ being normalized version of x(t). Bottom - signal $z_\Phi(t)$ presented in angular domain determined by $\varphi(t)$.

Therefore, vibration signal produced by considered shaft at time t is be expressed as:

$$s(t,\dot{\varphi}(t),l(t)) = A_s(\dot{\varphi}(t),l(t))\sin\left(2\pi\int_{-\infty}^{\infty}\dot{\varphi}(t)\,dt\right) \qquad (5)$$

where $A_s(\cdot)$ denotes load and speed dependent amplitude for simulated harmonic component.

Two key elements are present in the proposed model. First, the resulting signal consist of frequency modulated component $\sin\left(2\pi\int_{-\infty}^{\infty}\dot{\varphi}(t)\,dt\right)$ determined by given angular speed $\dot{\varphi}(t) = \frac{d\varphi(t)}{dt}$. Second, amplitude-modulating function $A_s(\dot{\varphi}(t),l(t))$ of two time-dependent variables, namely rotational frequency $\dot{\varphi}(t)$ and load $l(t)$.

3.2 Rolling element bearings - GATD signal

Signal *x(t)* is defined to be generally angular-temporal deterministic for given phase increments $\varphi(t)$ with the initial phase φ_0 (synchronization angle) and angular period Φ

when:

$$z_{\mathrm{T}}(\varphi_0 + \varphi(t+\tau)) = z_{\mathrm{T}}(\varphi_0 + \varphi(t+\tau) + n\Phi), \qquad (6)$$

for every τ ranging between $0 < \tau \leq T$ and $n \in \mathbb{Z}$. Where $\frac{1}{T} = max(\frac{d\varphi(t)}{dt})$,

where $z_{\mathrm{T}}(t)$ is normalized (z-scored) version of signal *x(t)* for the angular period Φ and synchronization angle φ_0:

$$z_T^{\Phi}(\mathrm{t}) = \frac{x(\mathrm{t}) - \mu_T^{\Phi}(t)}{\sigma_T^{\Phi}(t)}, \qquad (7)$$

and $\sigma_T^{\Phi}(t)$ being localized angular standard deviation of *x(t)* for the period T, angular period Φ and phase increments $\varphi(t)$:

$$\sigma_T^{\Phi}(t) = \sqrt{\frac{1}{\mathrm{T}} \int_{u=t(\varphi_0+m\Phi)}^{t(\varphi_0+m\Phi)+\mathrm{T}} \left(x(u) - \mu_T^{\Phi}(u)\right)^2 du}, \qquad (8)$$

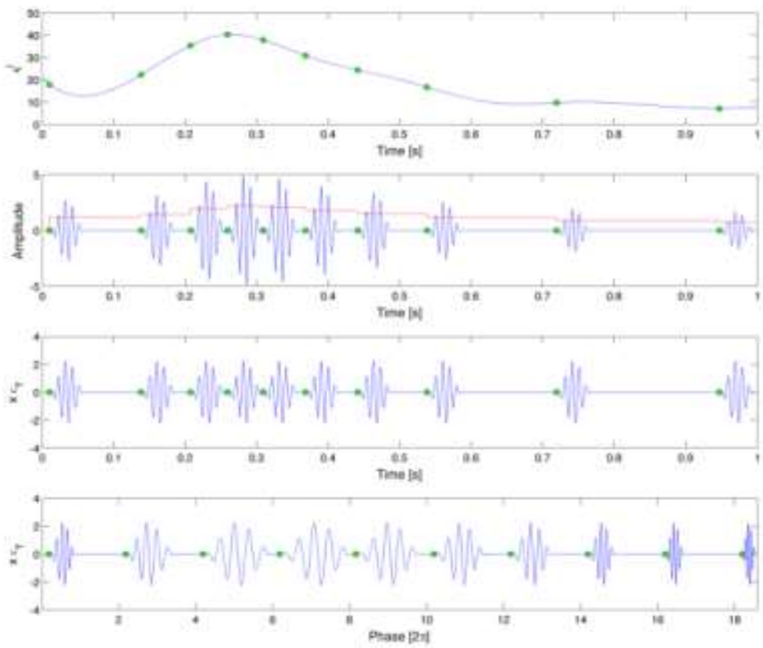

Fig. 4. The concept of general angular-temporal signal. Top panel – instantaneous frequency $\dot{\varphi}(t) = \frac{d\varphi(t)}{dt}$. Top/middle panel – exemplary signal x(t) (blue) and corresponding localized angular-temporal standard deviation $\sigma_T^{\Phi}(t)$ (red). Bottom/middle panel – normalized signal z(t). Bottom panel – normalized signal $z_T^{\Phi}(\varphi(t))$

for $m = \left\lfloor \frac{\varphi(t)-\varphi_0}{\Phi} \right\rfloor$ and $\mu_T^{\Phi}(t)$being a localized angular mean of x(t) for the time period T and angular period Φ :

$$\mu_T^{\Phi}(t) = \frac{1}{T}\int_{u=t(\varphi_0+m\Phi)}^{t(\varphi_0+m\Phi)+T} x(u)\,du. \qquad (9)$$

Intuitively, angular temporal deterministic signal consists of series of repetitions of the same temporal pattern equally spaced in angular domain. In addition, for GATD signals standard deviation of such pattern might vary between the consecutive cycles defined in in joint angular-temporal domain as positioned at angle $\varphi_0 + m\Phi$ and lasting for time period T.

Following aforementioned assumption, the model for vibration signals produced by operating faulty rolling element bearing is proposed and expressed by:

$$b(t;\dot{\varphi}(t),l(t))=\sum_{n=-\infty}^{\infty}A(t(n\Phi);\dot{\varphi}(t),l(t))w(t-t(n\Phi-\varphi_0)-\tau_n), \qquad (10)$$

where *w(t)* is a single acoustic wave generated when rolling element strikes damaged surface, Φ is the characteristic period for REB fault and $\mathrm{A}(\cdot)$ denotes load and speed dependent amplitude for the simulated signal and τ is a set of independent identically distributed random variables responsible for simulating rolling elements jitter effect. It can now be noticed that when assuming constant speed and load conditions, $\dot{\varphi}(t)=const$ and $l(t)=const$, resulting $\mathrm{A}(\cdot)=const$ as well, and the proposed model takes the form of:

$$b(\varphi)=\sum_{n=-\infty}^{\infty}A\mathrm{w}(\varphi-n\Phi-\varphi_0-\tau_n) \qquad (11)$$

that follows the principles of cyclostationary modeling of operating rolling element bearing with localized outer race fault proposed by Antoni et al. (see eq. 32 in [7]).

4 Experiment - estimation of model coefficients

Signals acquired for each pair of operational conditions were then separated into deterministic and random parts using DRS algorithm proposed by Antoni et al. in [8]. The purpose of separation was to determine different speed/load amplitude characteristic for shaft related vibration components and for bearing related components, as former manifests themselves in periodic parts of the signal while later in a random one. It is to be stated that the purpose of the experiment was only to observe a general behavior of the system. In order to obtain speed/load characteristics of the particular harmonic component it is crucial to track the magnitude of each one separately as operational conditions changes.

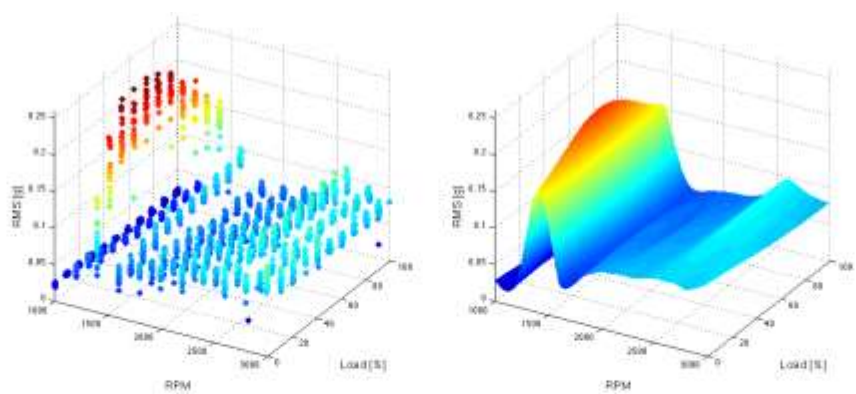

Fig. 5. Observed values for deterministic component of the signal (left). Estimated smooth surface (right).

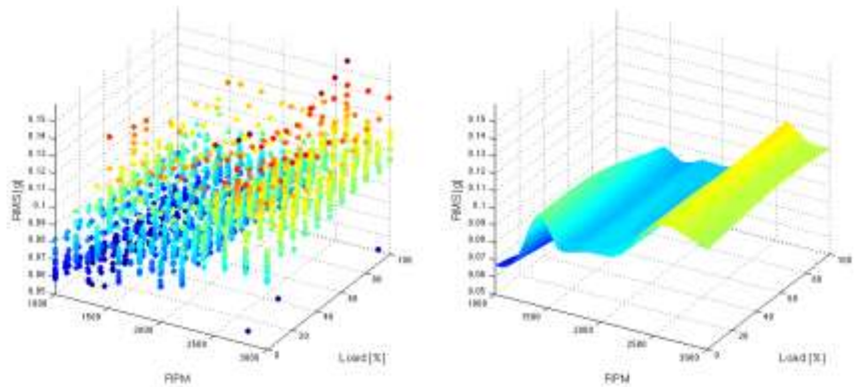

Fig. 6. Observed values for random component of the signal (left). Estimated smooth surface (right).

As a result, two sets of scatter data sets were obtained, as illustrated in fig. 5 and fig. 6, reflecting the amplitudes of both deterministic and random components of the signal as a function of speed and load. In order to obtain smooth characteristic that could then be used for modeling, the generalized additive models (GAM) is used to estimate the surface that the most precisely interpolates observed values in different operational conditions.

5 Example and discussion

To present the outcome of the proposed model we simulate the vibration signal consisting of fundamental of shaft related components with the characteristic frequency equal to $1.0 \cdot \dot{\varphi}(t)$ and one rolling element bearing with localized fault on the outer race with the characteristic frequency equal to $1.6 \cdot \dot{\varphi}(t)$. It is assumed that the response w(t) is an exponentially descending sine wave of the frequency centered around 4kHz (see fig. 7 and fig. 8). In order to simulate extremely varying operational conditions we assume linearly ascending speed profile from 1000 up to 3000 RPM and rapidly ascending load profile – as for example in the runup process of mechanical object.

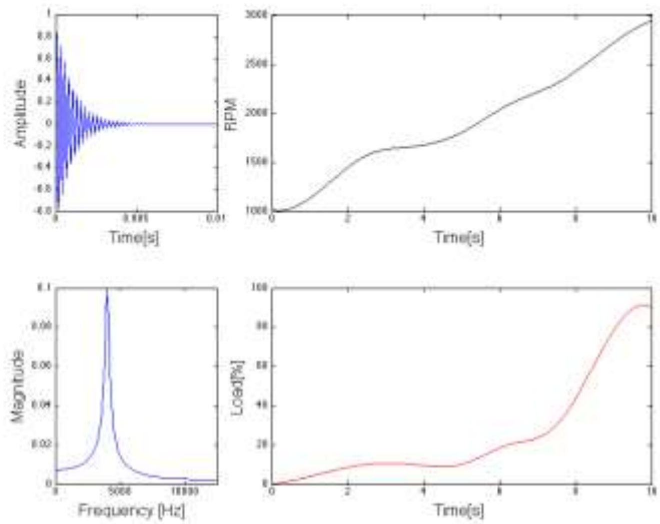

Fig. 7. Inputs for used for the simulation. Top left – time view of the pulse response excited by faulty bearing. Bottom left – spectrum of the pulse response excited by faulty bearing Top right – speed profile used for simulation. Bottom right – load profile used for simulation.

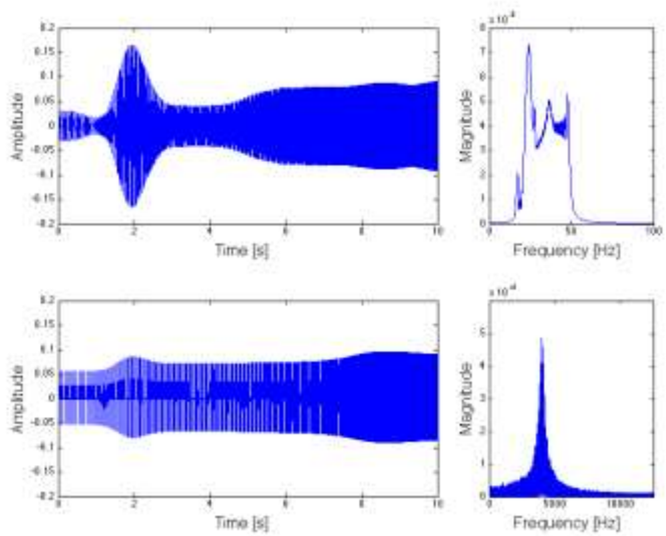

Fig. 8. Signals generated with the model. Top – shaft related component – time view and spectrum. Bottom – Rolling element bearing related component **-** time view and spectrum.

The presented new definition of vibration signals has enabled modeling of signals generated by machinery working under varying operational conditions. Fig. 8 clearly shows that the proposed model results in a well-described simulated signal, which included the expected speed-and-load dependence of components' amplitudes.

Acknowledgements

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