3.3 The concept of stationarity of real signals

Although commonly misinterpreted, the notion of signal "stationarity" is not unequivocally defined because it simply might not be. Considering following remark from [5]:

> *"(…) for stationarity, for measurements on a machine, are typically that [the machine] is operating at constant speed and load. If the function being averaged using the expectation operator E [·] is the signal itself, that is fx (t) = x (t), then the result of the average will be the mean value. If fx (t) = x2 (t), the result will be the mean square value."*

it is clear that the notion of stationarity is just a prose definition, i.e. described in words, and any mathematical formulation containing "const" notions are just generalizations.

3.4 Classification of signals – second approach

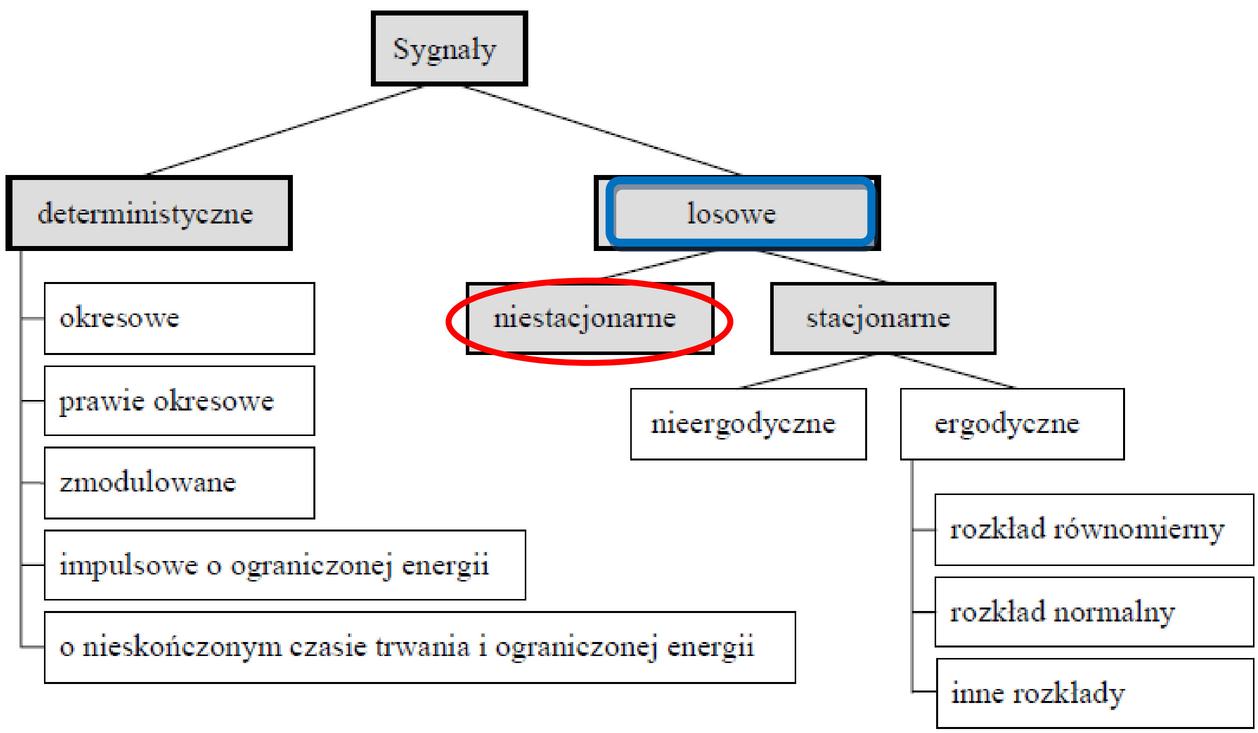

Figure 6. Classification according to Prof. T. Zieliński [2] (AGH, Polska)

Figures 5 and 6 show a different, yet equally accepted classification of vibration signals, actually of any signals. When classifying a vibration signal generated by any real object, or equally valid, recorded by any real equipment, it is of utmost importance to keep in mind that:

- such signals are inevitably contaminated with noise, which results in implementation of the Wold's decomposition theorem, which states that any stationary discrete-time stochastic process can be decomposed into a pair of uncorrelated processes, one deterministic, and the other being a moving average process:

$$x(n) = x_d(n) + x_l(n)$$

where:
$x(n)$ – vibration signal,
$x_d(n)$ – deterministic (random) part,
$x_d(n) = -\sum_{k=1}^{\infty} \propto_k x_d(n-k), \propto_k : predictions\ coefficients$
$x_l(n)$ – stochastic (random) part,

therefore, any real signal ought to be classified as random signal, which implies a stationary signal according to Prof. Randall's classification, which is clearly not true in terms of signals to be analyzed in the project.

- such signals are always represented by a smeared spectra, which implied that they are not stationary (or nonstationary), which excludes random signals according to Prof. Randall's classification,

- such signals might be modelled by cyclostationary signal analyses, which means that they are not limited to a probabilistic techniques, as is implied by Prof. Zieliński according to his classification.

3.5 The concept of cyclo-stationarity

In recen;ty years [3], another topology of non-stationary signals has been proposed by Prof. Jerome Antoni from University of Lyon, France, and is illustrated in Figure 7, taking advantage of the observation that rotating machine signals are characterized by inherent periodicities in the mechanisms that produce them because all kinematical variables in the machinery are periodic with respect to some rotation angles. This approach enabled formulation of new group of signals, for instance:

- cyclostationary signals,
- cyclostationary deterministic signals,

- quasi-periodic processes,
- a general group of non-stationary signals not characterized by any inner cycles.

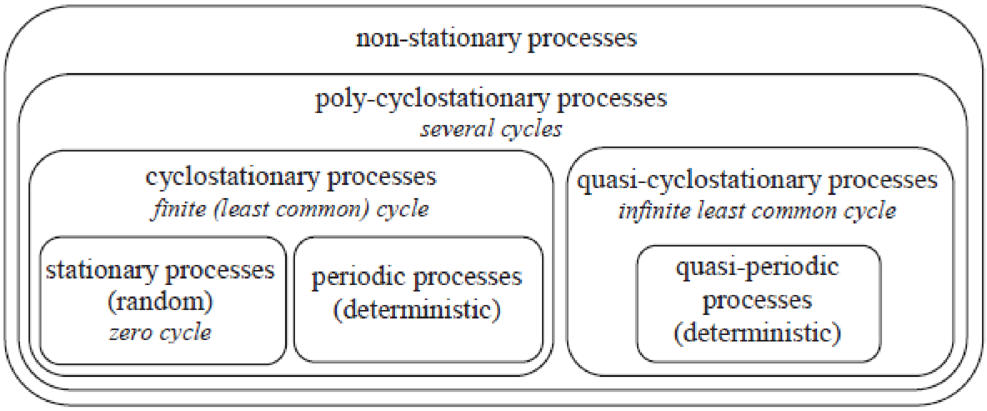

Figure 7. Classification of non-stationary signals [3]

This approach is the general approach accepted by the project team members, because it enables analysis of time-and-frequency varying signals without a classical fragmentation tools, which is basic accepted approach. In details, considering a state-of-the-art approach [1], it is assumed that:

> "*In condition monitoring, changes in vibration signals are ascribed to changes in condition, so it is important that other factors which cause changes in vibration signals are considerably reduced or eliminated. Vibrations tend to change with the speed and load of a machine, so (…) the signals generated by a rotating machine operating at constant speed and load [*are typically analyzed*], for which the signals will typically be stationary and/or cyclostationary. Occasionally, use can be made of non-stationary signals, such as those generated by a machine under run-up or coast-down conditions, but such signals should be processed with the appropriate analysis techniques, such as the time/frequency techniques* (…)"

From the quote above, two simple conclusion might be drawn:

- typical analyzed signals should be free from fluctuation of process parameters
- signals associate with fluctuation of process parameters should be analyzed with time-frequency tools EXPLICITELY.

At this point, it is very important to recall that the aim of the project is to extend the current classification of vibration signals in order to overcome these limitations.

4 Proposition of a novel class of vibration signals

Within the project specifications, the process of a definition of a class of signals consists of two separate parts, namely proposition of a novel class, which is done in the current chapter, followed by formulation of mathematical description, which is obviously more valuable, and is carried in the next task of the project.

Within the current task, the proposition includes:

- description of motivation for definition of a novel class
- description of variable operational parameters,
- specification of necessary, external information required for class definition,
- proposition of the name of the class.

Some kinds of machinery inherently work in quickly changing operational parameters. This group of machines include wind turbines, mining machinery, excavators, etc. The concept of data analysis of machinery under varying operational conditions has been illustrated in works [4-12]. The literature states that operational parameters have significant influence on the frequency contents of vibration signals. As Zimroz stated in [10]:

> "(…) *considering the wide variation in operating conditions (…) and the dependence between the operating conditions and the diagnostic features, there is a need to take the operating conditions (…) into account during the reasoning process. If the operating conditions are neglected, one may obtain an unclear basis for diagnostic decision taking due to the crossing effect between the good condition and the bad one on a diagnostic feature distribution.*"

Therefore, the team members claim that a strong need for definition and mathematical formulation of a class of signal describing this particular scenario.

Consider a real variation of process parameters illustrated in figure 8.

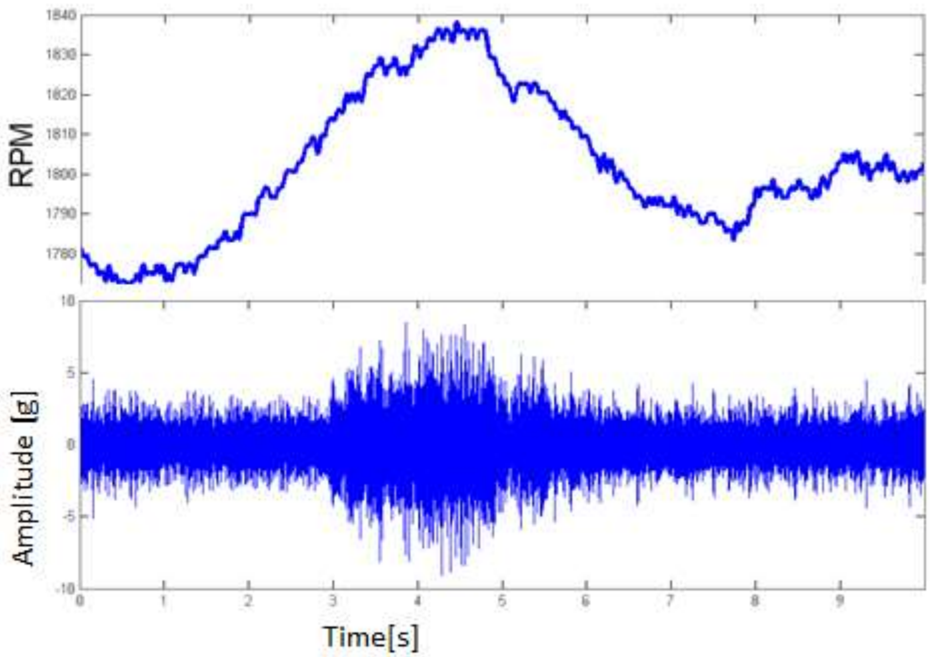

Figure 8. Illustration of real variation of process parameters of a wind turbine

Figure 8 illustrates a variation of rotational speed of a wind turbine. The speed is a major parameter, which influences the vibration contents from rotary machinery next to load. In order to develop and test new condition-monitoring methodology it is important to simulate signals generated by machinery operating under non-stationary operational conditions.

In this work we propose the following mathematical model of vibrations x(t) generated by rotor machinery operating under extremely varying operational conditions.

$$x(t;\varphi(t),l(t)) = s(t;\varphi(t),l(t)) + b(t;\varphi(t),l(t)) + n(t)$$

where

$$s(t;\varphi(t),l(t))$$

denotes the component generated by rotating shafts and

$$b(t; \varphi(t), l(t))$$

corresponds to rolling element bearing related vibrations. Additionally, n(t) denotes Gaussian noise.

In order to simulate non-stationary vibration signal originated from machinery operating under highly varying regime following information are necessary:

- Instantaneous rotational speed of the machine $\varphi(t)$ (fig.9)
- Instantaneous load $l(t)$ (fig. 10)
- Amplitude given as a function of two variables, namely: speed and load denoted as $A_b\big(t; \dot{\varphi}(t), l(t)\big)$ (fig. 11)

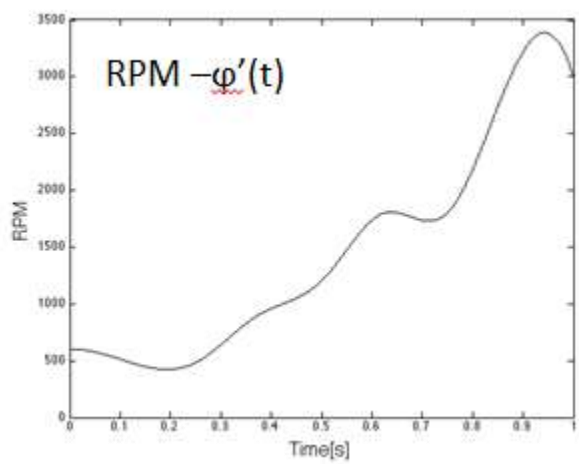

Figure 9. A profile of simulated speed

Next, on the basis of this profile, a speed-load profile is generated, and illustrated in figure 11. This characteristics is convolved with a second-order time signal illustrated in figure 12.

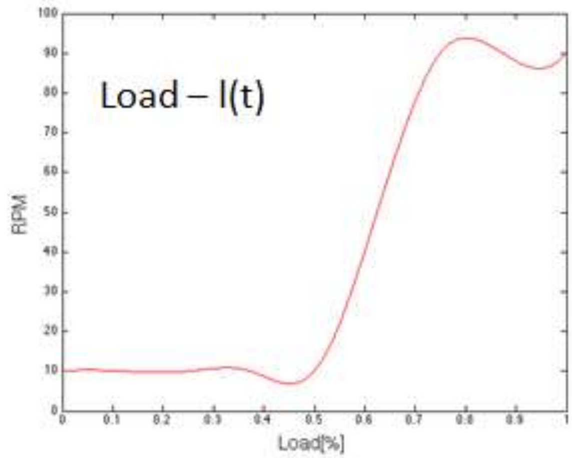

Figure 10. A profile of simulated load

As mentioned earlier in this chapter **proposed class of signals** encapsulates signals, which contain **two** sorts of components:

- amplitude modulated angle-deterministic signals,
- amplitude modulated angular/temporal-deterministic signal.

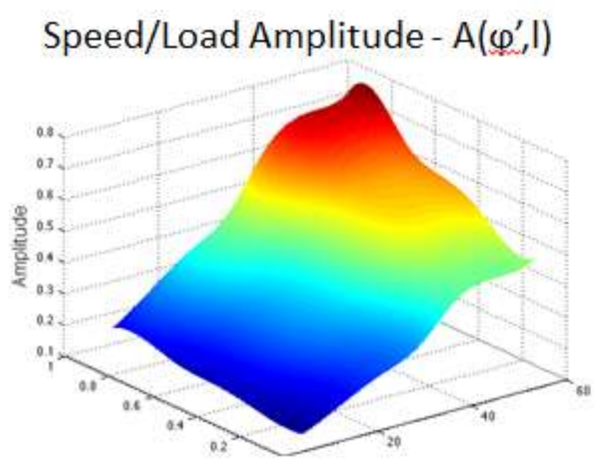

Figure 11. Speed and load dependent amplitude function

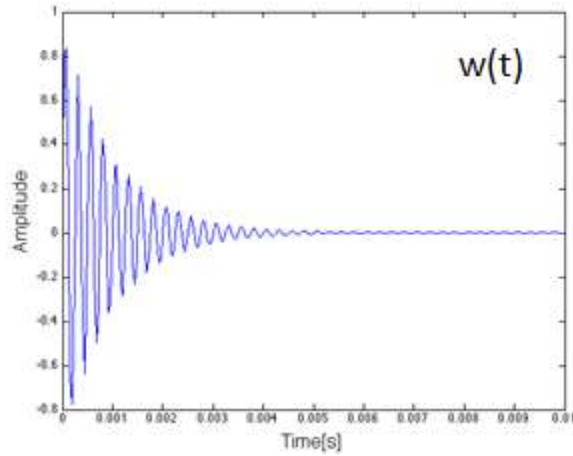

Figure 12. Illustration of an exemplary impulseresponse

The first group of component are **deterministic** frequency components, which are phase-locked to the shaft speed, and amplitude of which is specified by abovementioned speed-load function. and can be expressed by:

$$s(t;\varphi(t),l(t)) = A_s(t(\varphi);\dot{\varphi}(t),l(t))\sin\left(2\pi\int_{-\infty}^{\infty}\dot{\varphi}(t)\,dt\right)$$

where:

$$\dot{\varphi}(t) = \frac{d\varphi(t)}{dt}$$

An exemplary generation of such components is illustrated in figure 13.

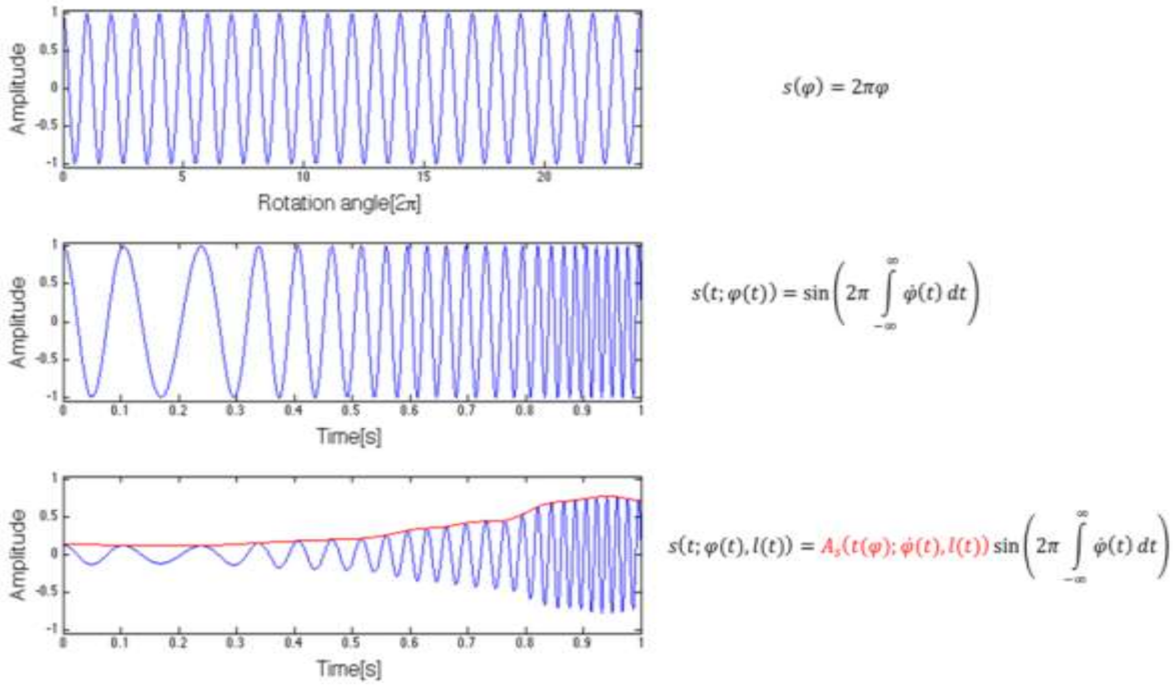

Figure 13. Generation of deterministic phase-locked components. Top: a stationary signal, center: a phase-modulated signal, bottom: a amplitude modulated angle–deterministic signal

The top panel of figure 13 represent typical sinusoidal wave; however introduced in the angular domain (phase-locked). The same signal is presented on the middle panel, but this time in time-domain. As it can be seen the signal is no longer periodic. Additionally, on the bottom panel we present discussed signal amplitude-modulated by function related to instantaneous speed and load. Resulting signal is now considered to be **amplitude modulated angle – deterministic.** Meaning that it consist of the component deterministic in angular domain but further amplitude modulated by external function.

The second group of components are **random, quasi-periodic** frequency components, which are still phase-locked to the shaft speed, and amplitude of which is specified by abovementioned speed-load function. Proposed tytpe of signals is given by:

$$b(t;\varphi(t),l(t)) = A_b\big(t;\dot{\varphi}(t),l(t)\big) \sum_{k=-\infty}^{\infty} \mathrm{w}(\mathrm{t} - \mathrm{t}(\mathrm{k}\Phi) - \tau)$$

where:

w(t) – single acoustic wave generated when rolling element strikes damaged surface (fig. 12)

Φ – characteristic angular period for REB fault

A generation of such components is illustrated in figure 14.Such components are typical for rolling-element bearings (REB) encountered in rotary machinery.

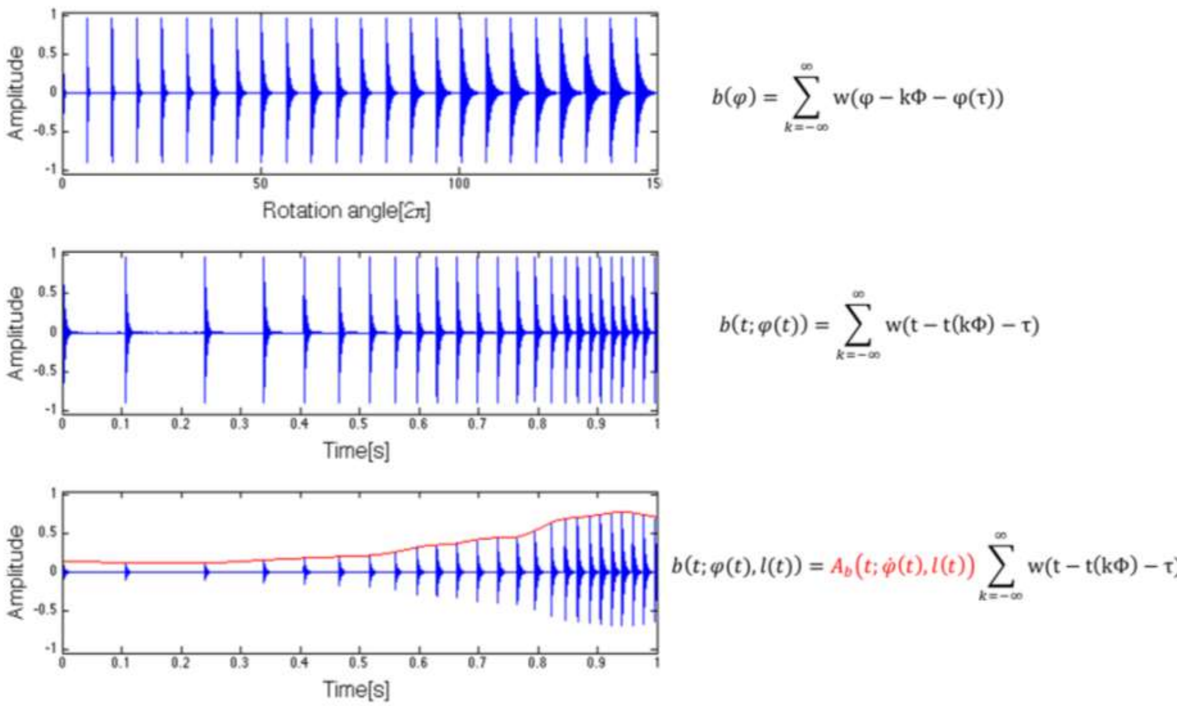

Figure 14. Generation of random, quasi-periodic phase-locked components. Top: a cyclo-stationary signal, center: a random, quasi-periodic phase-modulated signal, bottom: amplitude modulated angular/temporal – deterministic signal.

The top panel of figure 14 presents the example of angular/temporal deterministic signal plotted in the angular domain. The same signal is presented on the bottom panel in time domain. It can be seen that discussed signal consist of sum of signals corresponding to single acoustic wave generated by faulty