difficult. Situation is quite different for impulse signal. All of the pulses can be recognized well recognized in time domain. The only obstacle is the scales axis instead of frequency. That is a serious practical obstacle that makes interpretation of the scalogram difficult.

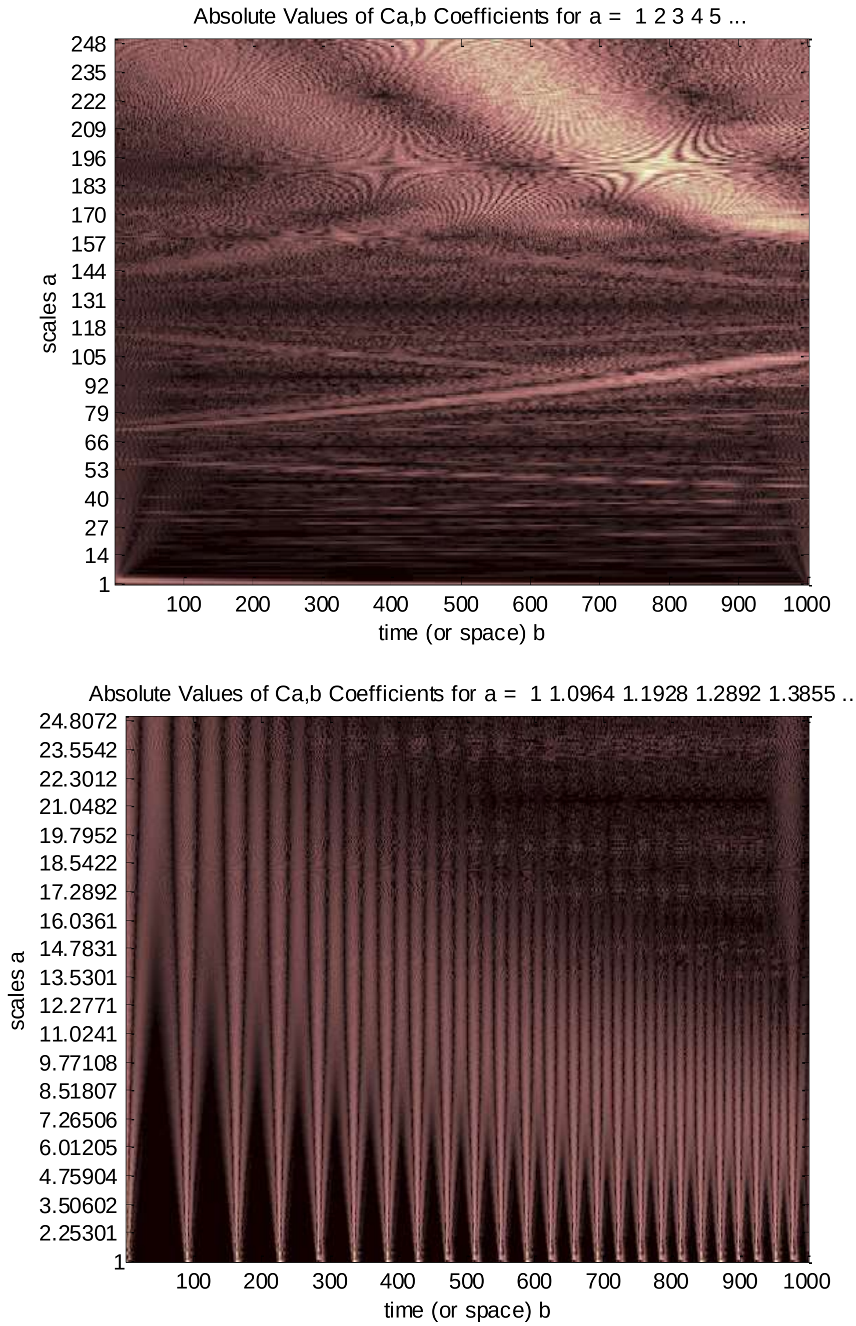

Fig.5. Scalograms of test signals. Top – linear chirp, bottom – pulse signal.

Unfortunately, many studies using wavelet analysis have suffered from an apparent lack of quantitative results. The wavelet transform has been regarded many as an interesting diversion that produces colorful pictures, yet purely qualitative results. This misconception is in some sense the fault of wavelet analysis itself, as it involves a transform from a one-dimensional time series (or frequency spectrum) to a diffuse two-dimensional time–frequency image. This diffuseness has been exacerbated by the use of arbitrary normalizations and the lack of statistical significance tests.

4 Instantaneous frequency estimation

Problem of the Instantaneous Frequency (IF) estimation is a fundamental issue in signal processing [22, 23]. As a generalization of the definition of frequency, IF is defined as the rate of change of the phase angle at time t of the analytic version of the signal. Analytical form of the signal is given by the following expression:

$$z(t) = x_r(t) + jx_u(t), \tag{3}$$

The mathematical instrument to obtain the imaginary part of the analytic signal is the Hilbert transform:

$$x_u(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} x_r(\tau)\frac{1}{t-\tau}d\tau, \tag{4}$$

Using the feature that the imaginary part of exponential representation of a complex number is its phase, the phase angle can now be obtained using the equation below:

$$\varphi(t) = imag(\log z(t)), \tag{5}$$

Then IF can be easily defined as a:

$$f(t) = \frac{d\varphi(t)}{dt}, \tag{6}$$

There are a few interesting approaches dedicated for accurate estimation of IF successfully applied in the nonstationary signal processing.

One of the most fundamental and robust methods is the one that bases on Hilbert transform [24] as presented in the definition above. Even though this approach uses directly the definition of IF, results obtained from real signals are usually affected with serious estimation error when dealing with relatively complicated signals and low signal-to-noise ratios. Therefore, there is a necessity for improved estimation of IF.

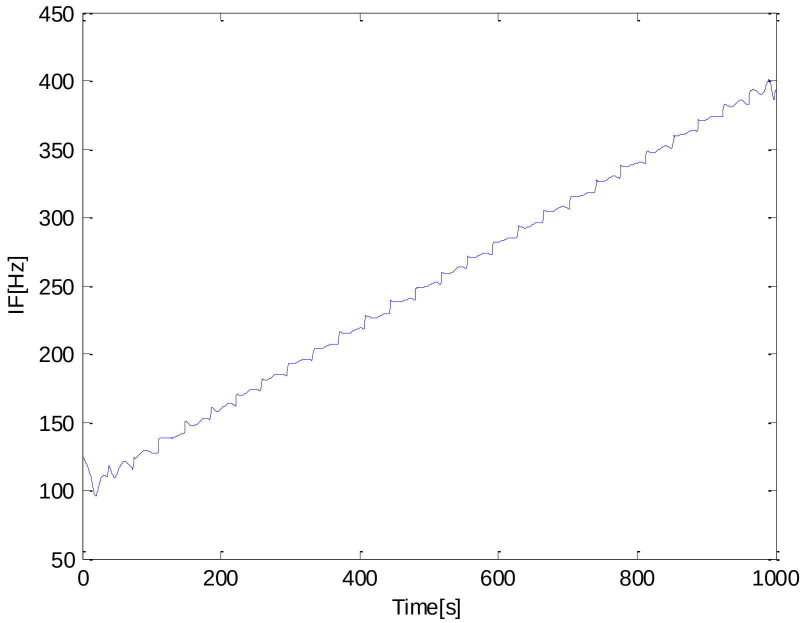

Fig.6. Instantaneous frequency of tested chirp signal estimated by narrow-bad phase demodulation method.

Ref. 25 presents time-scale based method that is capable of estimation of relative changes of IF not the IF itself. However, presented methods can return satisfactory results for relatively complex multi-component signals.

Another approach that is to use time-frequency representation of the signal for estimation of IF. The time-frequency based method appears to be robust to energy changes of the selected components [26]. Moreover, it can deal with significant speed variations even for relatively complex signals. Unfortunately, the accuracy of the method is strongly affected by frequency resolution of the spectrogram. Due to uncertainty principle, influence of this disadvantage is inversely proportional to the length of the analyzed signal.

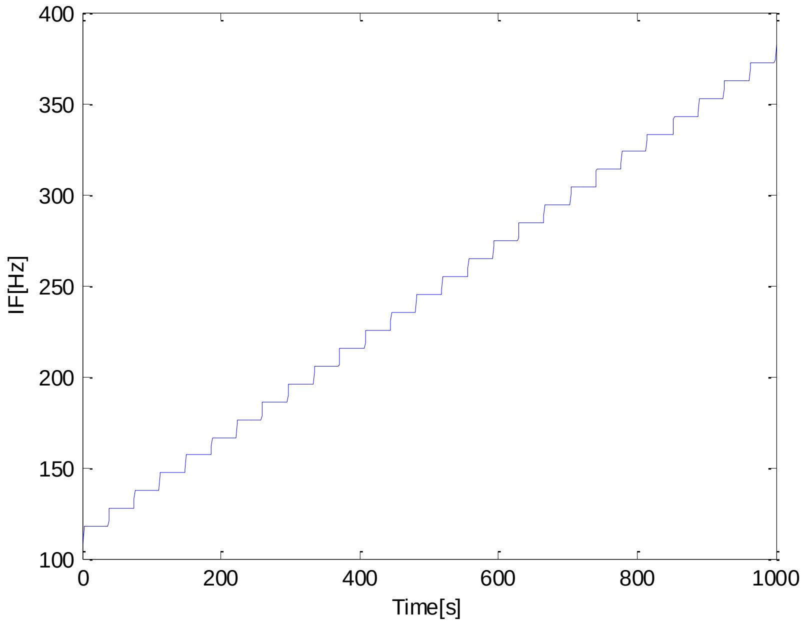

Fig.7. Instantaneous frequency of tested chirp signal estimated by time-frequency.

Figures 6 and 7 presents IF estimated from chirp test signal. Because of the random character of a pulse signal, tests were performed using only the chirp signal. As seen on both figures, Described methods were able to estimate the IF of the test signal. In the figure 6 the influence of narrow-band filtration can be seen as a slight variations of the resulted IF. Time-frequency based method also allowed good estimation of the IF. This time however, resulting IF was affected by limited frequency resolution of time-frequency representation used for calculation.

Another approaches that are worth mentioning in this review are methods based on Teager-Kaiser techniques [27] and on polynomial modeling [28] of sought IF function. The algorithm is based on the Teager-Kaiser nonlinear differential energy operator and leads to the calculation of the instantaneous amplitude and frequency of the response. Compared to the Hilbert transform, it leads at least to the same estimation errors, but in addition presents smaller computational complexity and faster adaptation, due to its instantaneous nature. By using a polynomial function instead of the linear chirp kernel in the chirplet transform, the polynomial chirplet transform can produce a time-frequency distribution with excellent concentration for a wide range of signals with a continuous instantaneous frequency.

5 Empirical Mode Decomposition

During last decade a novel method called empirical mode decomposition (EMD) is being popularized for nonstationary signals analysis. Using the EMD method, any complicated data set can be decomposed into a finite and often small number of components, which is a collection of intrinsic mode functions (IMF). An IMF represents a generally simple oscillatory mode as a counterpart to the simple harmonic function. By definition, an IMF is any function with the same number of extrema and zero crossings, with its envelopes being symmetric with respect to zero. It is worth to notice that there is no mathematical expression describing the IMF.

An IMF is defined as a function that satisfies the following requirements:

- In the whole data set, the number of extrema and the number of zero-crossings must either be equal or differ at most by one.
- At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

Therefore, an IMF represents a simple oscillatory mode as a counterpart to the simple harmonic function, but it is much more general: instead of constant amplitude and frequency in a simple harmonic component, an IMF can have variable amplitude and frequency along the time axis. The procedure of extracting an IMF is called sifting. The sifting process is as follows:

1. Identify all the local extrema in the test data.
2. Connect all the local maxima by a cubic spline line as the upper envelope.
3. Repeat the procedure for the local minima to produce the lower envelope.

The upper and lower envelopes should cover all the data between them. Their mean is m1. The difference between the data and m1 is the first component h1:

$$X(t) - m_1 = h_1 .$$

Ideally, h1 should satisfy the definition of an IMF, for the construction of h1 described above should have made it symmetric and having all maxima positive and all minima negative. After the first round of sifting, a crest may become a local maximum. New extrema generated in this way actually reveal the proper modes lost in the initial examination. In the subsequent sifting process, h1 can only be treated as a proto-IMF. In the next step, it is treated as the data, then

$$h_1 - m_{11} = h_{11}.$$

After repeated sifting up to k times, h1 becomes an IMF, that is

$$h_{1(k-1)} - m_{1k} = h_{1k}.$$

Then, it is designated as the first IMF component from the data:

$$c_1 = h_{1k}.$$

The stoppage criterion determines the number of sifting steps to produce an IMF. Two different stoppage criteria have been used traditionally. Huang proposes the first criterion. It similar to the Cauchy convergence test, and we define a sum of the difference SD as

$$SD_k = \frac{\sum_{t=0}^{T} |h_{k-1}(t) - h_k(t)|^2}{\sum_{t=0}^{T} h_{k-1}^2(t)}.$$

Then the sifting process is stop when SD is smaller than a pre-given value.

A second criterion is based on the number called the S-number, which is defined as the number of consecutive siftings when the numbers of zero-crossings and extrema are equal or at most differing by one. Specifically, an S-number is pre-selected. The sifting process will stop only if for S consecutive times the numbers of zero-crossings and extrema stay the same, and are equal or at most differ by one. Once a stoppage criterion is selected, the first IMF, c1, can be obtained. Overall, c1 should contain the finest scale or the shortest period component of the signal. We can, then, separate c1 from the rest of the data by $X(t) - c_1 = r_1.$ Since the residue, r1, still contains longer period variations in the data, it is treated as the new data and subjected to the same sifting process as described above. This procedure can be repeated to all the subsequent rj's, and the result is

$$r_{n-1} - c_n = r_n.$$

The sifting process stops finally when the residue, rn, becomes a monotonic function from which no more IMF can be extracted. From the above equations, we can induce that

$$X(t) = \sum_{j=1}^{n} c_j + r_n.$$

Thus, a decomposition of the data into n-empirical modes is achieved. The components of the EMD are usually physically meaningful, for the characteristic scales are defined by the physical data.

This decomposition method operating in the time domain is adaptive and highly efficient. Since the decomposition is based on the local characteristic time scale of the data, it can be applied to nonlinear and nonstationary processes.

As seen in fig.8 even linear chirp signal can be decomposed into intrinsic mode function using EMD. Because the method is nonparametric and it lacks of proper mathematical description, obtained results are difficult for interpretation. Additional quantitive description is also impossible.

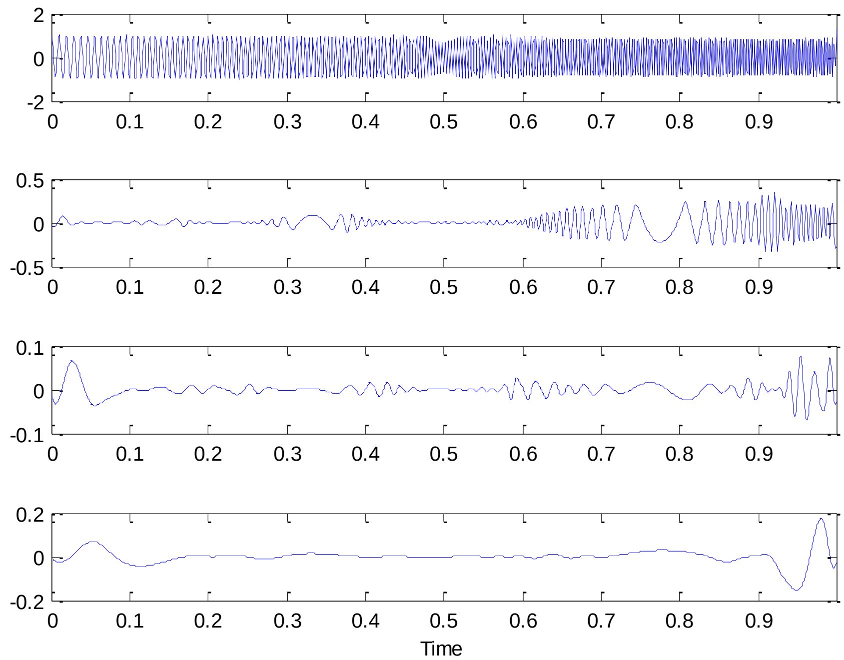

Fig.8. Chirp signal decomposed into intrinsic modes.

For impulse signal situation is similar. EMD algorythm was able to decompose the signal into a set of modes. However, due to the fact that tested signal cosist only of a non-stationary component, the results are unlikely do be useful.

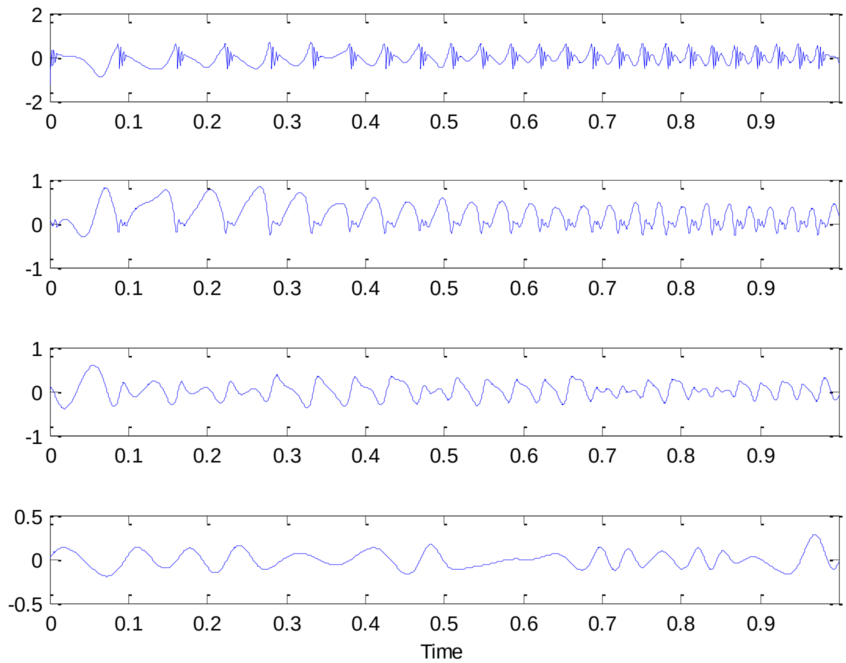

Fig.9. Pulse signal decomposed into intrinsic modes.

Although EMD found itself extremely useful in the terms of nonstationary signals processing [30] even more accurate identification is possible using so called Hilbert–Huang transform that uses results of EMD as an input for Hilbert spectrum [31]. It can be noticed that IMF can serve as an input data for many other methods like support vector machine [32] or neural networks [33].

6 Cyclostationarity-based approach

Pure research on cyclostationarity in mechanical systems signal processing had been started in late 90's [34]. Although, cyclostationarity as an approach to the signal processing dates back to the early sixties, it is truly since the eighties that it has become a subject of the active research [34]. This approach was first used in the field of telecommunication in order to model the process of signal modulation. Science then, it has become popularized especially in the field of communication and financial analysis. In those days, William A. Gardner in his handbook “Cyclostationarity in telecommunication and signal processing” [35-40] foresaw many other fields of application of cyclostationarity, including mechanical vibrations analysis.

The application of cyclostationarity to signal processing allows obtaining many useful information, including: detection of random components, classification of multiple received signals present in noisy data set according to their modulation types, estimation of signal parameters, prediction of a future behavior of the random signal etc...

Definition 1:"*Signal is cyclostationary of order n (in the wide sense) if and only if we can find some n'th-order nonlinear transformation of the signal that will generate finite-strength additive sine-wave components which result in spectral lines. For example, for n=2, a quadratic transformation (like the squared signal or the product of the signal with delayed version of itself, or the weighted sum of the such products) will generate the spectral lines."* [35].

This definition allows to distinct several orders of cyclostationarity depending on the order of non-linear transformations involved. That means that any cyclostationary behavior that can be detected by n-th degree non-linear transformation can be defined as n-th order cyclostationarity.

According to definition 1, for a signal that exhibits first-order cyclostationarity, first-degree transformation can create a periodic component. First-degree transformation is in fact linear transformation, so a signal that exhibits first-order cyclostationarity is simply a periodic signal. Subsequently, signal exhibits second-order cyclostationarity if any quadratic transformation (e.g. squaring) generates a periodic component.

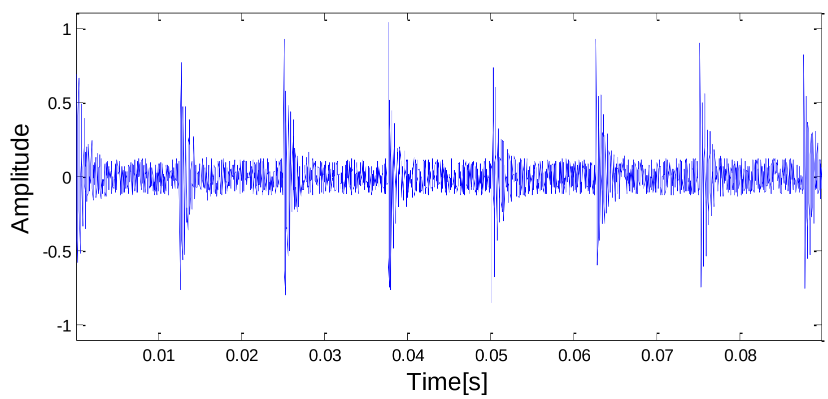

Fig.10. Time view of the exemplary second-order cyclostationary signal [41].

One of the most fundamental, yet powerful tools for cyclostationarity-based analysis is spectral correlation density. Spectral correlation density can be defined as:

$$SC_x^{\alpha}(f) = \lim_{\Delta f \to 0} \lim_{\Delta t \to \infty} \frac{1}{\Delta t} \int_{-\Delta t/2}^{\Delta t/2} \Delta f X_{1/\Delta f}(t, f + \alpha/2) \cdot X^*_{1/\Delta f}(t, f - \alpha/2) dt \quad , \tag{3}$$

where $X_{1/\Delta f}(t,v)$ is the complex envelope of filtered version of the signal x(t) in a narrow frequency-band [v-Δf/2; v+Δf/2],

$$X_{1/\Delta f}(t,v) \overset{\Delta}{=} \int_{t-1/2\Delta f}^{t+1/2\Delta f} x(u)e^{-j2\pi vu}du \quad , \tag{4}$$

and it can be interpreted as an Short-Time Fourier Transform calculated for rectangular time window [22]. Alternatively to equation 3, formula for spectra correlation can be given by:

$$SC_x^{\alpha}(f) = \lim_{\Delta f \to 0}\lim_{T\to\infty}\frac{1}{T\Delta f}\int_T x_{\Delta f}(t, f+\tfrac{\alpha}{2})x_{\Delta f}^{*}(t, f-\tfrac{\alpha}{2})e^{-j2\pi\alpha t}dt \quad , \tag{5}$$

where xΔf(t; f) stands for the filtered version of x(t) in a narrow frequency-band [f-Δf/2;f+Δf/2]. At this point it should be mentioned that expression $e^{-j2\pi\alpha t}$ in formula 5 performs frequency shifting operation (down-conversion), by cyclic frequency α in order to provide the same central frequency for components $x_{\Delta f}(t, f+\tfrac{\alpha}{2})$ and $x_{\Delta f}(t, f-\tfrac{\alpha}{2})$. On the other hand, since operation presented in equation 4 can be interpreted as a joint down-sampling and filtering procedure, down-conversion operation takes place during calculation of $X_{1/\Delta f}(t,v)$.

It could be noticed that exemplary signal presented in fig.10 was similar to impulsive test signal used in this chapter. The only difference was that for the test signal, pulses were unequally spaced while for the exemplary signal period of pulses occurrence was equal. Therefore, spectral correlation density was calculated for the impulsive test signal used in this chapter. As it is shown in figure 11 not many information can be obtained from resulting plots. It is caused by the fact that analyzed signal even though was non-stationary, was not cyclostationary.

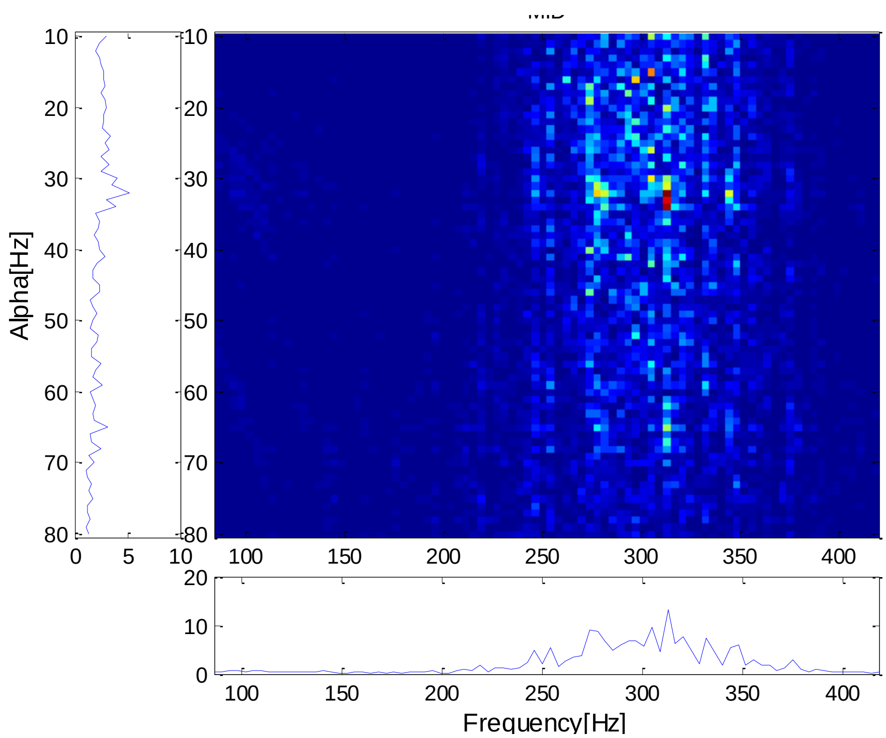

Fig.11. Spectral correlation density of tested pulse signal.

Nowadays, it can be observed that cyclostationarity-based models can be expanded to more general class. For example, signals where both cycle length and cycle energy are changing due to external governing function [42, 43]. Two main concepts can be distinguished. Namely: cyclo-non-stationarity [44] and fuzzy cyclostationarity [43]. The concept of fuzzy-cyclostationarity assumes that cyclic frequency of observed process is itself phase-modulated by a periodic function. Even though, fuzzy-cyclostationarity is one step towards generalization of cyclostationary processes its application to the subject of the following project is highly limited.

Although, there is no formalization of the definition of cyclo-non-stationary process, based on information published so far it can be assumed that such class of processes might be treated as a generalization of cyclostationarity. In general, cyclo-non-stationary process, like cyclostationary, is periodically correlated; however the period of correlation is expressed not in the time domain but as a angular phase of governing function. Therefore, proper definition of the class of cyclo-non-stationary processes will be the main subject of this research project.

7 Conclusion

In the presented task number of general approaches to non-stationary signals processing was analyzed. The purpose was to select the most promising methods that could be further used for analysis vibration signals generated by machinery operating under extremely varying operational conditions. Analyzed methods were divided into five subgroups. Namely:

- Time-frequency methods
- Wavelets
- Instantaneous frequency tracking
- Empirical Mode Decomposition
- Cyclostationarity

Among all analysed methods two main sub-groups were selected as a potential direction for further development. Namely: Instantaneous frequency tracking and cyclostationarity based approach. IF tracking is very important part of overall non-stationary signal analysis. Moreover, techniques used for estimation of IF might be used for reconstruction of rotational speed based on vibration signal itself. This approach will be investigated in the following research tasks of this project.

Another approach used for further development is cyclostationary analysis. It has proven its usefulness for identification and extraction of random components. It is one of the goals of this project to define generalized class of random processes in which cyclostationarity would be a sub-group. Based on presented references [42-45] it can be assumed that such definition is possible and that defined class of processes will allow better understanding of the nature of vibration signals generate by machinery operating in extremely varying operational conditions.

The realization of task No.2 will expand the research to analysis of diagnostic methods, i.e. paths of signal processing of rotating machinery working under non-stationary operational parameters.

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